Topology change in string theory

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Beyond Part III
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Outline

1. Basics
   - Calabi-Yau geometry
   - Type IIB String Compactification
2. Topology change in Type II
   - Physics of the conifold point
   - Resolving the conifold
   - Summary
3. The heterotic string

Definitions and properties 1

There are several ways to define Calabi-Yau 3-folds (CY3s). If X is a CY3, then:

- X is complex, and admits Ricci-flat Kähler metrics.
- Its holonomy is contained in $SU(3)$. This means it admits covariantly constant spinors.
- There exists on X a nowhere-vanishing holomorphic three-form Ω.
- $c_1(X) = 0$

Definitions and properties 2

The cohomology groups of a CY3 take a restricted form. The Hodge numbers are

$$\begin{matrix} & & & h^{00} & & & \\ & & h^{10} & & h^{01} & & \\ & h^{20} & & h^{11} & & h^{02} & \\ h^{30} & & h^{21} & & h^{12} & & h^{03} \\ & h^{13} & & h^{22} & & h^{31} & \\ & & h^{23} & & h^{32} & & \\ & & & h^{33} & & & \end{matrix} = \begin{matrix} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{11} & & 0 & \\ 1 & & h^{21} & & h^{21} & & 1 \\ & 0 & & h^{11} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{matrix}$$

Deformations

- There are two types of continuous *deformation* of a CY3:
  - Kähler deformations, parametrised by harmonic $(1,1)$-forms.
  - Complex structure deformations, parametrised by harmonic $(2,1)$-forms.
- Each independent deformation gives a massless scalar in the low-energy theory.
- Changing the geometry of the internal space X corresponds to changing the VEVs of the moduli fields.

Moduli space 1

- The Kähler structure can be parametrised by integrating the Kähler form over two-cycles:

$$t^i = \int_{C^i} K$$

where K is the Kähler form and $\{C^i\}$ a basis of two-cycles.

- The complex structure can be parametrised by the **periods** of Ω:

$$z^a = \int_{A^a} \Omega \qquad \mathcal{G}_a = \int_{B_a} \Omega$$

where $\{A^a, B_a\}$ is a symplectic basis of $H_3(X)$.

Moduli space 2

Focus on the complex structure:

- It turns out that $\mathcal{G}_a = \frac{\partial \mathcal{G}}{\partial z^a}$, where $\mathcal{G}$ is a certain holomorphic function.
- The moduli space is Kähler, with Kähler potential

$$\mathcal{K} = -\log\left[i(\bar{z}^a \frac{\partial \mathcal{G}}{\partial z^a} - z^a \frac{\partial \overline{\mathcal{G}}}{\partial \bar{z}^a})\right]$$

Mathematical result:
The resulting curvature blows up when e.g. $z^1 = 0$, but this locus is at a finite distance.

IIB string on a Calabi-Yau 3-fold

Type IIB strings on $\mathcal{M}_4 \times X$ yield $\mathcal{N} = 2$ SUGRA in $4D$ with

- $h^{11}(X) + 1$ hypermultiplets, containing the Kähler moduli.
- $h^{21}(X)$ vector multiplets, containing the complex structure moduli.

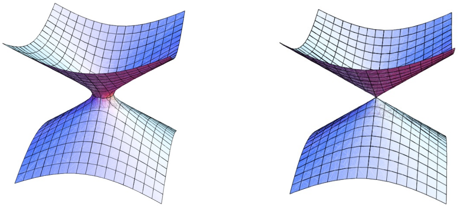
The moduli fields' kinetic terms are given by the metric on moduli space i.e.

$$\mathcal{L} = G_{\alpha\overline{\beta}}(z)\, \partial^{\mu} z^{\alpha} \partial_{\mu} \overline{z}^{\beta}$$

Topology change in Type II string theory

The conifold singularity 1

- Varying the complex structure of X can cause it to develop singularities.
- The simplest example corresponds to shrinking a three-cycle $T \cong S^3$ to a point.



The conifold singularity 2

- Suppose the homology class of T is A^1. Then, from our definition,

$$z^1 = \int_{A^1} \Omega = \int_T \Omega = 0$$

- Moduli space is singular at $z^1 = 0$ (in particular, $G_{1\bar{1}} \to \infty$).
- Therefore our low-energy theory ceases to make sense. What has gone wrong?

String theory fixes the conifold!

We have neglected some light charged degrees of freedom.

- A D3-brane can wrap around the three-cycle T, and appear as a $4D$ point particle. The 'kinetic term' in its action is:

$$S_K \sim \int_{T\times\mathbb{R}} \mathrm{Vol} = \int_T \mathrm{Vol} \int d\tau \to 0$$

- This is the action for a point particle, whose mass vanishes as $\mathrm{Vol}(T) \to 0$. (More rigorous: BPS condition.)

The new massless states

- The other term in the action is the coupling to the four-form potential:

$$S_C = \int_{T \times \mathbb{R}} C^{(4)}$$

- This leads to the particle being charged under the $U(1)$ gauge field superpartner of z^1.
- The resulting corrections to the beta-function repair the singularity on moduli space.

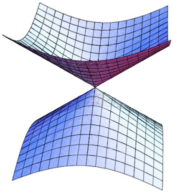
(Details in **Strominger: hep-th/9504090)**

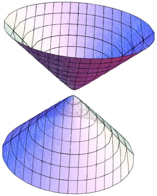
Things get more interesting if multiple three-spheres degenerate.

- Suppose N three-spheres, satisfying M homology relations, collapse to zero volume.
- Thus we get N light hypermultiplets charged under $N - M$ of the $U(1)$'s.
- This gives $N - M$ D-term constraints on N hypermultiplets $\Rightarrow$ there are flat directions.

(Details in **Greene et. al.: hep-th/9504145v2**)

- Taking VEVs along the flat directions Higgses the $N - M$ $U(1)$'s.
- Therefore the number of massless vector multiplets changes.
- Since this number is h^{21}, a topological invariant of a CY3, the topology of the background must have changed!



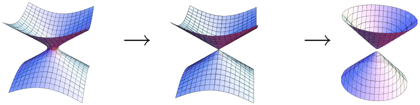


The collapsed three-spheres are blown up as two-spheres.

Conifold transitions in Type II string theory

In summary:

- Moduli space contains points where the CY3 becomes singular.
- Branes wrapping collapsing cycles become massless at these points, and their interactions smooth the physics.
- In favourable circumstances, these fields can condense, and geometrically this corresponds to a transition to a topologically distinct spacetime.



The heterotic string

Differences to Type II compactifications

The heterotic string theories have the following properties:

- CY3 compactification leads to $\mathcal{N} = 1$ SUSY in 4D.
- All scalars therefore fall into chiral multiplets.
- The only branes in heterotic theories are the NS5 branes
- They contain 10D Yang-Mills fields, therefore extra data is required for compactification: a stable, holomorphic vector bundle V on the CY3.

Supersymmetry and branes

- $\mathcal{N} = 1$ SUSY imposes fewer restrictions on quantum corrections to the moduli space.
- There are no branes which can wrap cycles to give $4D$ particles.

Heterotic anomaly condition/Bianchi identity

- Green-Schwarz anomaly cancellation requires an unconventional Bianchi identity for the three-form field strength H:

$$dH = Tr[F \wedge F - R \wedge R]$$

  where F is the Yang-Mills field strength and R the Ricci tensor.
- $dH = 0$ then becomes a topological condition on the vector bundle V:

$$c_2(V) = c_2(X)$$

Transgression

- So part of the story must involve taking vector bundles across conifold transitions, while maintaining the anomaly condition.
- For a recent discussion, see **Candelas et. al. arXiv:0706.3134**

Summary

- Non-perturbative charged states become massless at certain points in type II moduli space.
- These states can condense and realise a change in the topology of spacetime.
- The analogue of this process in the heterotic string is not yet understood.