Axions and string theory

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Outline

The strong CP problem and axions

The Green-Schwarz mechanism

String axions

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The θ term

- To QCD (or any 4D gauge theory) we can add a topological term:

$$S_{\text{top.}} = -\int d^4x \frac{\theta}{32\pi^2} \epsilon^{\mu\nu\rho\tau} \text{tr}(F_{\mu\nu} F_{\rho\tau}) = -\int \frac{\theta}{8\pi^2} \text{tr}(F \wedge F)$$

- This has the following properties:
  - $S_{\text{top.}}/\theta$ evaluates to the instanton number.
  - The coefficient θ is therefore periodic: $\theta \sim \theta + 2\pi$
  - $S_{\text{top.}}$ is odd under P and CP.
- Physically, $S_{\text{top.}}$ leads to CP-violating effects such as an electric dipole moment for the neutron. Experiments: $|\theta| \lesssim 10^{-10}$.

The θ term and quark masses

- General quark mass terms take the form

$$\sum_f \left(m_f q^\dagger_{Lf} q_{Rf} + m^*_f q^\dagger_{Rf} q_{Lf}) \right)$$

- We can make all m_f real by a chiral field rotation:

$$\left. \begin{array}{l} q_{Lf} \longrightarrow e^{-\mathrm{i}\alpha_f} q_{Lf} \\ q_{Rf} \longrightarrow e^{\mathrm{i}\alpha_f} q_{Rf} \end{array} \right\} \quad \Rightarrow \quad m_f \longrightarrow e^{2\mathrm{i}\alpha_f} m_f$$

- But via the chiral anomaly, this redefinition also changes θ:

$$\theta \longrightarrow \theta + 2 \sum_f \alpha_f$$

- So the physical parameter is actually $\bar{\theta} := \theta - \sum_f \arg(m_f)$

Solving the strong CP problem

- Suppose that $m_u = 0$.
- A chiral rotation of q_u can then set $\overline{\theta} = 0$ with no other consequences.
- But $m_u = 0$ apparently strongly disfavoured by lattice results.

A simple axion model

- A different solution is required. To the Standard Model, add:
  - a new pair of left- and right-handed quarks, ψ_L, ψ_R.
  - a singlet scalar ϕ.
- Impose a chiral symmetry $U(1)_{\mathcal{A}}$
  $\psi_L \to e^{-\mathrm{i}\sigma}\psi_L \ , \ \psi_R \to e^{\mathrm{i}\sigma}\psi_R \ , \ \phi \to e^{-2\mathrm{i}\sigma}\phi$.
- The Lagrangian is

$$\mathcal{L} = \mathcal{L}_{SM} + \mathrm{i}\overline{\psi}\gamma^\mu D_\mu\psi + \partial^\mu\phi^*\partial_\mu\phi + \lambda\big(\phi\,\psi_L^\dagger\psi_R + \phi^*\psi_R^\dagger\psi_L\big) - V(\phi)$$

A simple axion model

$$\mathcal{L} = \mathcal{L}_{SM} + \mathrm{i}\overline{\psi}\gamma^\mu D_\mu \psi + \partial^\mu \phi^* \partial_\mu \phi + \lambda\big(\phi\, \psi_L^\dagger \psi_R + \phi^* \psi_R^\dagger \psi_L\big) - V(\phi)$$

- Break $U(1)_{\mathcal{A}}$ by a VEV for ϕ. Let $\langle \phi \rangle = F_a/\sqrt{2}$, and expand ϕ:

$$\phi = e^{-\mathrm{i}\varphi}\big(\frac{F_a}{\sqrt{2}} + \rho\big)$$

- Immediate consequences:
  - ψ and ρ get masses $\sim F_a$, and decouple.
  - φ is the Goldstone boson of $U(1)_{\mathcal{A}}$ breaking, so classically massless.
  - φ is dimensionless, with kinetic term $\frac{F_a^2}{2}\partial^\mu \varphi\, \partial_\mu \varphi$.

The $U(1)_{\mathcal{A}}$ anomaly

- At low energies, only φ transforms under $U(1)_{\mathcal{A}}$, $\varphi \to \varphi + 2\sigma$.
- But $U(1)_{\mathcal{A}}$ transformations induce $\overline{\theta} \to \overline{\theta} + 2\sigma$ via the anomaly.
- Deduce that $S_{\text{top.}}$ gets replaced in effective theory by

$$S_{\text{top.}} \to -\int \frac{1}{8\pi^2}(\overline{\theta} + \varphi)\mathrm{tr}(F \wedge F)$$

- Restore canonical kinetic term, $\varphi := a/F_a$.
- So low energy theory contains scalar a whose only non-derivative interaction is

$$-\frac{a}{32\pi^2 F_a}\epsilon^{\mu\nu\rho\tau}\mathrm{tr}(F_{\mu\nu}F_{\rho\tau})$$

The axion mass

- The shift symmetry $a \to a + \text{const.}$ is broken by instantons.
- Find effective potential from path integral.

$$V(a) \sim \Lambda_{\text{QCD}} \cos(\bar{\theta} + \frac{a}{F_a})$$

- Minimum is at $a = -F_a\bar{\theta}$. Theta term dynamically set to zero!
- Careful calculation gives the axion mass

$$m_a \sim \frac{F_\pi m_\pi}{F_a}$$

So for large F_a, axion is very weakly interacting and very light.

Experimental constraints

- If F_a too small, axions copiously produced by stars etc.
  - Observed cooling rates imply $F_a \gtrsim 10^9$ GeV.
- If F_a too large, too much axionic dark matter.
  Roughly: Hubble expansion means $a \sim$ const. until $H \lesssim m_a$. After this, a starts oscillating, and energy density decreases as $1/R^3$, like cold matter.
  - Measured dark matter abundance implies $F_a \lesssim 10^{12}$ GeV.

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10D supergravity + Yang-Mills

- 10D SUGRA contains a two-form B:
  - Field strength $H = dB$, analogous to $F = dA$ for Abelian gauge field.
  - Kinetic term is

$$S_{\text{kin}} = \int d^{10}x \left(-\frac{(\alpha')^2}{16\kappa_{10}^2} |H|^2 \right)$$

  - The 'Bianchi identity' $dH = 0$ is automatic from definition.

10D supergravity + Yang-Mills

- Coupled to Yang-Mills theory, things change:
    - H must now satisfy a modified Bianchi identity

$$dH = -\text{tr}(F \wedge F) + \dots$$

    - Therefore define $H = dB - \omega_Y + \dots$, where

$$\omega = \text{tr}(A \wedge dA + \frac{2}{3} A \wedge A \wedge A) \;\Rightarrow\; d\omega_Y = \text{tr}(F \wedge F)$$

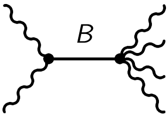
- H must be gauge invariant, but $\delta\omega_Y = \text{tr}(d\Lambda \wedge dA)$. Demand then that $\delta B = \text{tr}(d\Lambda \wedge A)$.

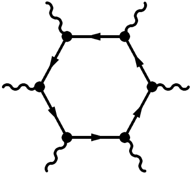
The Green-Schwarz mechanism

- We now have an interaction term containing B and two gluons

$$|H|^2 = -2dB \cdot \omega_Y + \dots$$

- String theory also contains an interaction $B \wedge \mathrm{tr}(F \wedge F \wedge F \wedge F)$. This is not gauge-invariant, since $\delta B \neq 0$.
- Together these terms give a tree-level process which cancels the hexagon anomaly





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Papers

- Following is largely based on:

  Axions in string theory, hep-th/0605206
  P. Svrcek and E. Witten

- QCD axion versus moduli stabilisation in string theory:

  The QCD axion and moduli stabilisation, hep-th/0602233
  J. Conlon

The ‘model-independent’ axion

- Let spacetime be $M_4 \times X$, X a compact 6-manifold.
- In 4D, impose the Bianchi identify via an auxiliary scalar a

$$S = \int d^4x \left(-\frac{(\alpha')^2 V_X}{16\kappa_{10}^2} |H|^2 \right) + \int a(dH - \mathrm{tr}(F \wedge F) + \ldots)$$

- It is now possible to integrate out H instead of a. We get

$$H = -\frac{8\kappa_{10}^2}{(\alpha')^2 V_X} * da$$

and the action becomes

$$S = \int d^4x \left(-\frac{4\kappa_{10}^2}{(\alpha')^2 V_X} |da|^2 - a\,\mathrm{tr}(F \wedge F) + \ldots \right)$$

Model-independent axion decay constant

- We see that $F_a = \frac{\kappa_{10}}{2\sqrt{2}\,\alpha'\pi^2\sqrt{V_X}}$. What is this in 4D terms?
- Dimensionally reduce the Einstein-Hilbert action to get

$$\int d^4x\,\sqrt{G}\,\frac{V_X}{2\kappa_{10}^2}R \;\equiv\; \int d^4x\,\sqrt{G}\,\frac{M_P^2}{2}R$$

- On the other hand, the Yang-Mills kinetic term gives

$$-\int d^4x\,\frac{\alpha' V_X}{8\kappa_{10}^2}\mathrm{tr}(|F|^2) \;\equiv\; -\int d^4x\,\frac{1}{2g_{YM}^2}\mathrm{tr}(|F|^2)+\ldots$$

- We therefore have

$$F_a = \frac{\kappa_{10}}{2\sqrt{2}\,\alpha'\pi^2\sqrt{V_X}} = \frac{M_P g_{YM}^2}{8\sqrt{2}\pi^2} \simeq 1.1\times 10^{16}\mathrm{GeV}$$

String theory fail?!

Not quite.

- Upper bound on F_a is for QCD axion, where $m_a \sim \frac{\Lambda_{\text{QCD}^2}}{F_a}$
- Model-independent axion couples to *all* gauge fields.
- Strongly-coupled hidden sector with $\Lambda_{\text{hid.}} \gg \Lambda_{\text{QCD}}$ gives large enough mass to a.

Model-dependent axions

- Equation of motion for B is just $\Box_X B = 0$. Solve

$$B = \sum_{i=1}^{b_4} b_i \, \omega_i$$

Here $\{\omega_i\}$ is a basis of harmonic 2-forms on X.

- Now consider the terms arising from Green-Schwarz mechanism:

$$\int B \wedge (F \wedge F \wedge F \wedge F) = \sum_i \int_X (\omega_i \wedge F \wedge F) \int d^4x \, b_i \, F \wedge F$$

- Kinetic term $|H|^2$ gives kinetic terms for b_i in $4D$:

$$S \sim -\int d^4x \sum_i \frac{(\alpha')^2 V_X^{\frac{1}{3}}}{16\kappa_{10}^2} |db_i|^2$$

- Similar to before, Svrcek and Witten get

$$F_b \sim 10^{17}\mathrm{GeV}$$

- Lesson: string theory likes large axion decay constants.

Summary

- Axions elegantly solve the strong CP problem.
- Experimental bounds restrict F_a to a narrow window.
- String theory gives axions naturally, but generally not a QCD axion.